#### Note to the Teacher





## Please read this before you duplicate the Student Workbook.



This Smart Skies™ FlyBy Math™ Student Workbook contains activities for one Air Traffic Control problem. In particular, the Workbook includes worksheets for 6 different calculation methods your students can use to solve the problem:

---Count feet and seconds

--- Draw and stack blocks

---Plot points on two vertical lines

---Plot points on a grid

- ---Use the distance-rate-time formula
- ---Graph two linear equations

You will most likely want to **assign only 1 or 2 of the calculation methods**. So we recommend that you follow these steps before you duplicate the Workbook.



#### 1. Select and Keep:

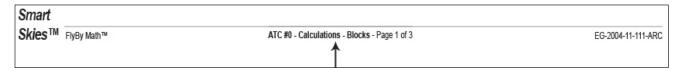
Choose the calculation method(s) you want your students to use.



#### 2. Recycle the Rest:

Remove the calculation worksheets you do not wish to assign.

To find the calculation worksheets, look for the Calculations footers at the page bottoms.





Other activity worksheets follow the Calculation pages. Be sure you remove the Calculation worksheets ONLY.

For more information about the Smart Skies™ workbooks, please see the Smart Skies™ teacher materials available online at

Smart
Skies™ FlyBy Math™

http://smartskies.arc.nasa.gov

FlyBy Math™ Workbook Cover Letter EG-2004-11-111-ARC



# Math & Science for Air Traffic Control



## Problem #2

- Aircraft are on merging routes
- Aircraft are traveling at the same speed
- Aircraft start at different distances from where the routes meet

# STUDENT WORKBOOK

Investigator:	
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## READ THE PROBLEM

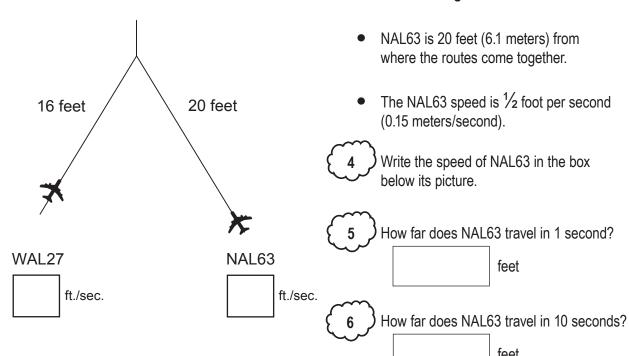




Begin Your Challenge: Will two planes, flying on merging jet routes, meet where the routes intersect?

#### Flight WAL27

- WAL27 is 16 feet (5.0 meters) from where the routes come together.
- The WAL27 speed is ½ foot per second (0.15 meters/second).
- Write the speed of WAL27 in the box below its picture.
- How far does WAL27 travel in 1 second? feet
- How far does WAL27 travel in 10 seconds? feet

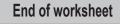


## To meet your Challenge, you will:

- · Conduct an experiment.
- · Do some math calculations.
- · Analyze your results.

## Then, you will use your results to answer these questions:

- Will the planes meet at the point where the routes intersect?
- If not, how many feet apart will the planes be when the first plane reaches the point where the routes intersect?



feet

feet

Flight NAL63



## **SET UP and DO THE EXPERIMENT**





Begin Your Task: Experiment to see what happens when the first plane reaches the point where the routes meet.

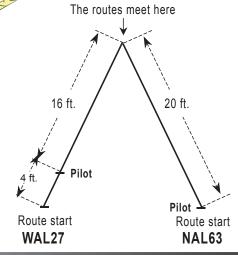
## Flight WAL27

- Begin at the point where the routes meet.

  Use chalk or masking tape to mark off a line 20 feet long.
  - The WAL27 pilot has a 4-foot headstart.
- Measure 4 feet from the route start and make a mark.

Place a "Pilot" label next to the mark.

## Lay out the jet routes.



#### Flight NAL63

- Begin at the point where the routes meet.

  Mark off a line 20 feet long.
- At the jet route start, make a mark.

  Place a "Pilot" label next to the mark.

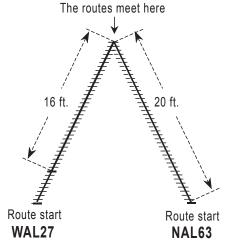
## ·<del>}</del>

## Flight WAL27

**5** Begin at the start of the jet route.

Place a mark (or piece of tape) every ½ foot (6 inches) along the jet route all the way to the point where the routes meet.

## Mark the speed control lines.



## Flight NAL63

Place a mark (or piece of tape) every

1/2 foot (6 inches) along the jet route all
the way to the point where the routes

meet.

Begin at the start of the jet route.



Investigator:		
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## **SET UP and DO THE EXPERIMENT**



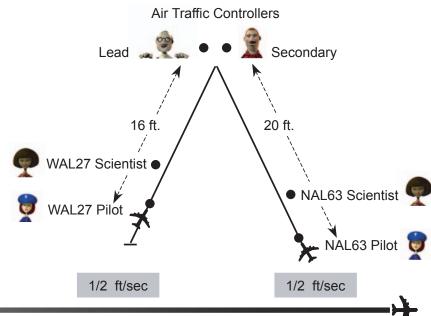


 $\binom{7}{7}$ 

Fill in this table with the starting conditions:

Flight Number	Speed (ft/sec)	Distance from the point where the routes meet (ft)
WAL27		
NAL63		

Circle your role on the route diagram on the right.





(9) [0]	Oo you think the pland where the routes com	es will meet at the point are together?	(10) Why or why not?	
Smart	Yes	No .		
<b>Skies</b> <sup>TM</sup>	FlyBy Math™	ATC #	2 - Experiment - Page 2 of 3	EG-2004-11-111-ARC

Investigator:		
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## **SET UP and DO THE EXPERIMENT**



 $\binom{11}{1}$ 

Circle your role and the steps below it.

Then, do the experiment 3 times using the steps for your role.

Do the experiment.

Steps	Lead Air Traffic Controller
Grobe	







**NASA Scientist** 

1 Take Your Position

Give the command: "Take your positions."

Go to your position on your jet route. Put one foot on each side of your route.

Go to your position.

Take your measuring tape to your jet route. Stand a few feet ahead of your pilot.

**2** Get Ready to Start

Give the command: "Ready."

Practice stepping down your route.

Take your first step on "1".

On each count, take a step

Wait for the first pilot to reach the point where the routes meet.

Move ahead of the pilot.

3 Start the Experiment

Start your stopwatch.
Count seconds: "1", "2", ...

When you hear "Halt", stop where you are.

to the next mark.

When the first pilot arrives, say "Halt". Record the seconds.

After you hear "Halt", measure and record the distance between pilots.

4 Stop the Experiment

When you hear "Halt", stop counting seconds.

Record the data.

Record the Halt Time measured by the Secondary Controller.

Record the Separation Distance measured by the NASA Scientist.

Use the results of your three experiments to choose the best answer to this question:

What is the separation distance where the routes meet?

Experiment:	1	2	3
Halt Time (seconds)			
Separation Distance (feet)			





## DO THE CALCULATIONS - Count Feet and Seconds





Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

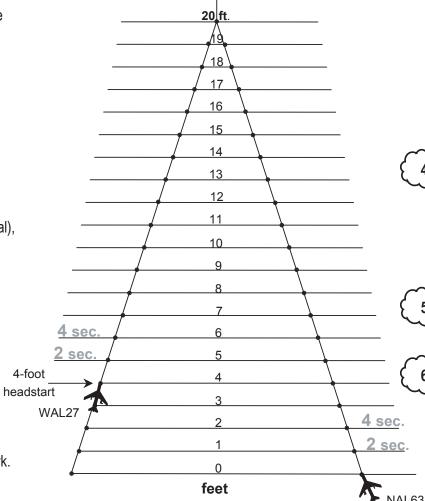
# Count feet and seconds along the jet route.

## Flight WAL27

- Starts 4 feet from the point where the route begins (a 4-foot "headstart").
- In 1 second, it moves  $\frac{1}{2}$  foot.
- In 2 seconds, it moves 1 foot from 4 feet to 5 feet.
- On the WAL27 jet route, find the 5-foot mark and trace "2 sec."
  - In the next 2 seconds (4 seconds total), WAL27 moves another foot from 5 feet to 6 feet.
- On the WAL27 jet route, find the 6-foot mark and trace "4 sec."
- Keep going on the WAL27 jet route, one foot at a time, until you reach the point where the routes meet.

At each foot-mark, write the total number of seconds to reach that mark.

4-foot



## Flight NAL63

- Starts where its route begins (at 0 feet).
- In 1 second, it moves \( \frac{1}{2} \) foot.
- In 2 seconds, it moves 1 foot from 0 feet to 1 foot.
- On the NAL63 jet route, find the 1-foot mark and trace "2 sec."
  - In the next 2 seconds (4 seconds total). NAL63 moves another foot from 1 foot to 2 feet.
- On the NAL63 jet route, find the 2-foot mark and trace "4 sec."
  - Keep going on the NAL63 jet route, one foot at a time, until you reach the point where the routes meet.

At each foot-mark, write the total number of seconds to reach that mark.

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Investigator:
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## **DO THE CALCULATIONS - Count Feet and Seconds**



# Answer the questions.

7	How many seconds did it take each plane to arrive at the point where the routes intersect?	WAL27	seconds	NAL63	seconds	
(8)	Did the planes meet at the point where the two routes into	ersect? Y	ES	NO		
(9)	If No, which plane arrived first? WAL27	NAL63				
(10)	How many seconds did it take this plane to travel to the p	oint where the two	routes intersect?		seconds	
(11)	At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane	traveled? How ma	feet iny feet was it from	the intersection?	)	
(12)	If you think two planes will meet, what would you tell the a	air traffic controller	to do to avoid a co	llision?		
(13)	You moved along each jet route, one foot at a time, to find Can you think of a faster way to find the number of secon			plane to travel to	the point where th	e routes meet.
Smart					End of	worksheet
<b>Skies</b> <sup>TI</sup>	FlyBy Math™ ATC	#2 - Calculations - C	ount - Page 2 of 2			EG-2004-11-111-ARC

## **DO THE CALCULATIONS - Draw Blocks**





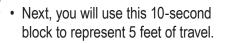
Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.



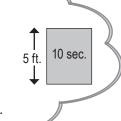
In this table, fill in the distance each plane will travel in 10 seconds.

The speed of each plane is  $\frac{1}{2}$  foot per second.

Each plane takes	1 second	2 seconds	4 seconds	10 seconds	
to travel	½ foot	1 foot	2 feet	fee	



· The block height represents 5 feet, the distance each plane travels in 10 seconds.



Fill in the number of seconds it will take each plane to travel the distance shown.

1 ft. 1 2 sec.

sec.

4 ft. sec.

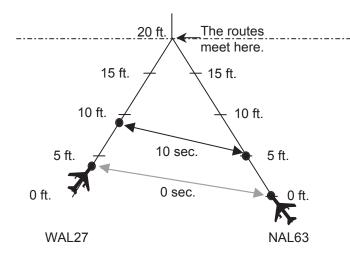


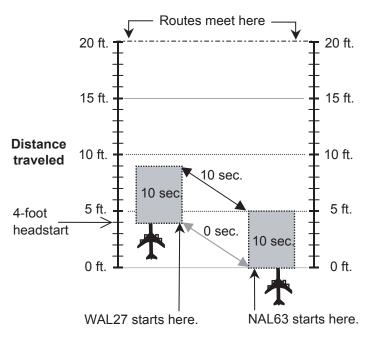


# Tollow along with this example of how to plot points and stack blocks.

## Flight WAL27

- Starts 4 feet from the point where the route begins (a 4-foot "headstart").
- In 10 seconds, it moves 5 feet.
- Now it is 9 feet along its route.
- Circle the 9-foot point on the WAL27 jet route.
- Trace the 10-second block for WAL27.





## Flight NAL63

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.
- Circle the 5-foot point on the NAL63 jet route.
- Trace the 10-second block for NAL63.

## **DO THE CALCULATIONS - Draw Blocks**

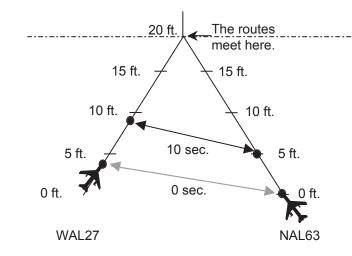


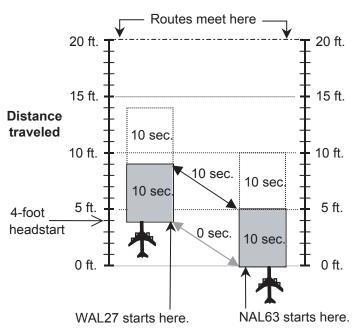


## Now it's your turn to draw and connect.

## Flight WAL27 and Flight NAL63

- On the route, draw a dot to show the position of WAL27 after 20 seconds.
- Trace the block on the graph below that shows the position of WAL27 after 20 seconds.
- On the route, draw a dot to show the position of NAL63 after 20 seconds.
- Trace the block on the graph that shows the position of NAL63 after 20 seconds.
- Connect your dots with a line marked "20 sec."
- Connect your blocks with a line marked "20 sec."





#### Now draw and connect at 30 seconds.

- Draw dots and blocks at 30 seconds.
- Connect dots and connect blocks at 30 seconds.

## Keep going...

Keep going until the first plane reaches the point where the routes meet. You may need to use a smaller block.

Be sure to use this smaller block for BOTH planes.

- How many feet long must the smaller block be? feet
- How many seconds does this block represent? secs

Smart

Investigator:	
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## **DO THE CALCULATIONS - Draw Blocks**



# Answer the questions.

(18)	Did the planes meet at the point where the two routes intersect?  YES NO	
(19)	If No, which plane arrived first? WAL27 NAL63	6
20	How many seconds did it take this plane to travel to the point where the two routes intersect?	seconds
(21)	At that time, how far away was the other airplane?  (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)	feet
(22)	If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?	

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## **DO THE CALCULATIONS - Plot Points on Lines**





Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

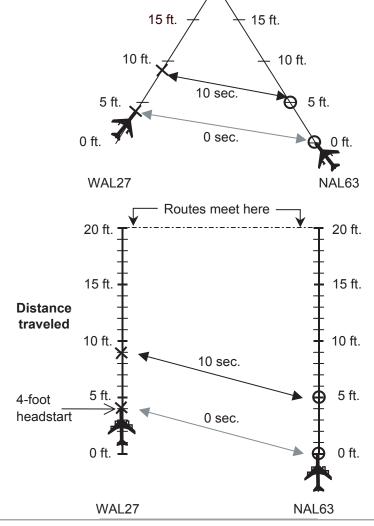
20 ft.

Follow along with this example of how to plot points.

The routes meet here.

## Flight WAL27

- Starts 4 feet from the point where the route begins (a 4-foot "headstart").
- In 10 seconds, it moves 5 feet.
- Now it is 9 feet along its route.
- Circle the **X** at the 9-foot point on the WAL27 jet route.
- Circle the **X** at the 9-foot point on the WAL27 line graph.



## Flight NAL63

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.
- Circle the **O** at the 5-foot point on the NAL63 jet route.
- Circle the **O** at the 5-foot point on the NAL63 line graph.

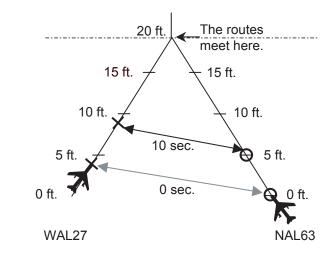
## **DO THE CALCULATIONS - Plot Points on Lines**

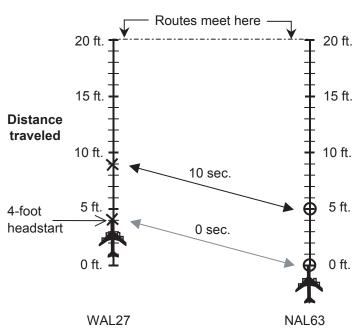




## Flight WAL27 and Flight NAL63

- On the route, draw an X to show the position of WAL27 after 20 seconds.
- On the line graph, draw an X to show the position of WAL27 after 20 seconds.
- On the route, draw an O to show the position of NAL63 after 20 seconds.
- On the line graph, draw an O to show the position of NAL63 after 20 seconds.
- On the routes, connect your X and **O** with a line marked "20 sec".
- On the line graph, connect your X and **O** with a line marked "20 sec".





## Now draw and connect at 30 seconds.

- On the routes, draw, connect, and label an X and an O at 30 seconds.
- On the graph, draw, connect, and label an X and an O at 30 seconds.

## Keep going...

- Keep going until the first plane reaches the point where the routes meet.
  - At its last step, the first plane may need to fly a distance shorter than 5 feet.

$\binom{14}{14}$	What is that shor	ter distance?
		feet

How many seconds does it represent? (Each plane travels 1 foot in 2 seconds.)

	seconds

How far does the second plane travel in that many seconds? feet

Investigator:
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## **DO THE CALCULATIONS - Plot Points on Lines**



	Answer the questions.
[17]	Did the planes meet at the point where the two routes intersect?  YES NO
18	If No, which plane arrived first? WAL27 NAL63
(19)	How many seconds did it take this plane to travel to the point where the two routes intersect?  seconds
20	At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)  feet
<u>[21]</u>	If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

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## DO THE CALCULATIONS - Plot Points on a Grid

20 ft.

15 ft.

10 ft.

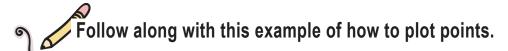
5 ft.

0 ft.





Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.



10 sec.

0 sec.

The routes

meet here.

10 ft.

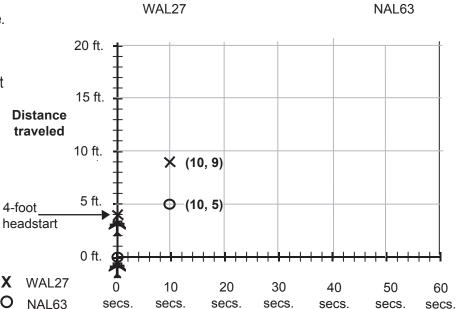
5 ft.

😧 0 ft.

15 ft.

## Flight WAL27

- Starts 4 feet from the point where the route begins (a 4-foot "headstart).
- In 10 seconds, it moves 5 feet.
- Now it is 9 feet along its route.
- Circle the **X** at the 9-foot point on the WAL27 jet route.
- Circle the **X** at the point (10, 9) on the grid.



### Flight NAL63

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.
- Circle the **0** at the 5-foot point on the NAL63 jet route.
- Circle the **O** at the point (10, 5) on the grid.

Smart Skies™ Time traveled

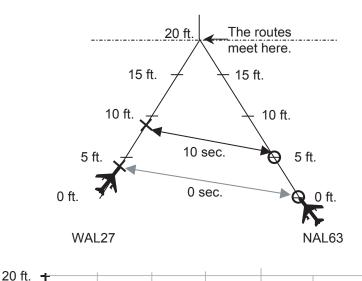
## DO THE CALCULATIONS - Plot Points on a Grid

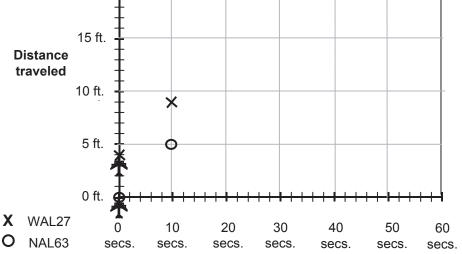




## Flight WAL27 and Flight NAL63

- On the route, draw an **X** to show the position of WAL27 after 20 seconds.
- On the grid, draw an **X** to show the position of WAL27 after 20 seconds.
- On the route, draw an **0** to show the position of NAL63 after 20 seconds.
- On the grid, draw an **O** to show the position of NAL63 after 20 seconds.
- On the routes, connect your **X** and **O** with a line marked "20 sec".





#### Now draw and connect at 30 seconds.

- On the routes, draw, connect, and label an **X** and an **O** at 30 secs.
- On the grid, draw an **X** and an **O** at 30 seconds.

## Keep going...

- Keep going until the first plane reaches the point where the routes meet.
  - At its last step, the first plane may need to fly a distance shorter than 5 ft.
- What is that shorter distance? feet
- How many seconds does it represent? (Each plane travels 1 foot in 2 secs.)

seconds

How far does the second plane travel in that many seconds?

feet

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Time traveled

## DO THE CALCULATIONS - Plot Points on a Grid



# Answer the questions

	Allowo	i tilo quootio	1101	
(17)	Did the planes meet at the point where the two routes intersect?  YES NO			
(18)	If No, which plane arrived first? WAL27 NA	L63		WAL27 ???
(19)	How many seconds did it take this plane to travel to the point where the two routes intersect?		seconds	
20	At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)		feet	
21	If you think two planes will meet, what would you tell the air traffic con	itroller to do to ave	oid a collision?	

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## DO THE CALCULATIONS - Use a Formula





Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.



The speed of each plane is 0.5 feet per second.



Use multiplication to fill in this table.

In this many seconds	Each p	olane travels t	his ma	any fe	eet			
1	0.5	feet/second	х	1	second	=	0.5	feet
2	0.5	feet/second	Х	2	seconds	=	1.0	foot
3	0.5	feet/second	Х	3	seconds	=	1.5	feet
4		feet/second	х		seconds	=		feet
5		feet/second	х		seconds	=		feet
6		feet/second	Х		seconds	=		feet

(	How could you use multiplication to find the distance each plane travels in 14 seconds?

- The pattern in the table suggests this rule:
   "To find the distance traveled, multiply the speed by the time traveled."
- In math and science, we often say "rate" instead of "speed."
- So we can write the rule like this:

 $distance = rate \times time$ 

- This relationship is called the Distance-Rate-Time Formula.
- We often write it like this:

Distance-Rate-Time Formula							
(	d	=	r	•	t		

(3) Use the formula to answer this question:

How many feet does each plane
travel in 20 seconds?

feet





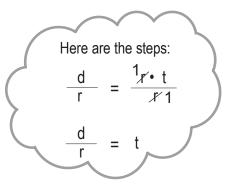
**Distance-Rate-Time Formula** 

$$d = r \cdot t$$

If we divide both sides of the equation by r...

...then, we get a formula for time traveled.

$$t = \frac{d}{r}$$



Use this formula to find the number of seconds for WAL27 to travel 16 feet to the point where the routes meet.

$$= \frac{16 \text{ feet}}{0.5 \text{ feet per second}} = \boxed{\text{seconds}}$$

Use the same formula to find the number of seconds for NAL63 to travel 20 feet to the point where the routes meet.

seconds

Will the planes meet at the point where the two routes intersect?

YES NO

7

If No, which plane will arrive first? WAL27

- NAL63 [
- How many seconds will it take this plane to travel to the point where the two routes intersect?
- At that time, how far away is the other airplane?

  (Hint: At that time, how many feet has the second plane traveled? How many feet is it from the intersection?)
- If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?



## **DO THE CALCULATIONS - Graph Linear Equations**





Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.



## Find an equation that describes the distance traveled by each

## Flight WAL27

We can use the Distance-Rate-Time formula

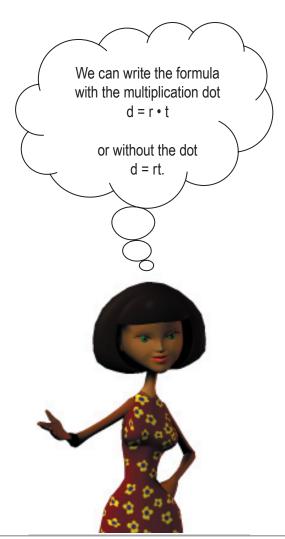
$$d = r \cdot t$$

to find d, the distance in feet WAL27 travels in t seconds.

- The WAL27 rate is 0.5 feet/second.
- WAL27 has a 4-foot headstart. (When you start your stopwatch at t = 0, WAL27 has already traveled 4 feet.)

So...

$$d = 0.5t + 4$$



## Flight NAL63

We can use the Distance-Rate-Time formula

$$d = r \cdot t$$

to find d, the distance in feet NAL63 travels in t seconds.

The NAL63 rate is 0.5 feet/second. So...

d = 0.5 t

## **DO THE CALCULATIONS - Graph Linear Equations**



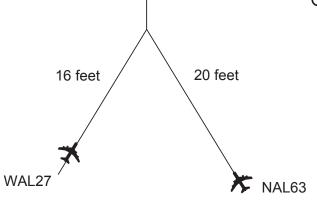


Fill in the table for WAL27.

$$d = 0.5t + 4$$

<b>t</b> seconds	<b>d</b> feet
0	
10	
20	
30	
40	



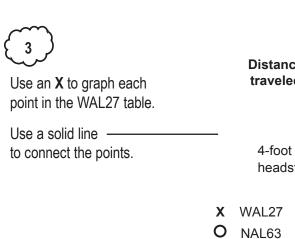


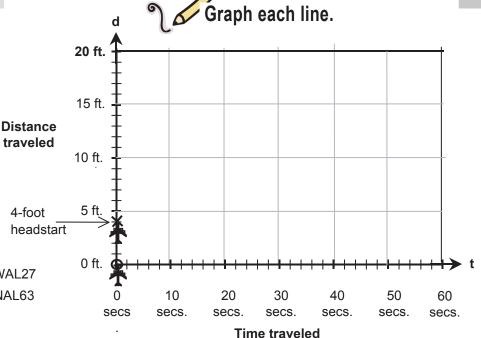
 $\binom{2}{2}$ 

Fill in the table for NAL63.

d = 0.5t

<b>t</b> seconds	<b>d</b> feet
0	
10	
20	
30	
40	







Use an **O** to graph each point in the NAL63 table.

Use a dotted line • • • • • • • • • to connect the points.



# Answer the questions.

, , ,	ny seconds will it take each p s intersect?	ane to arrive at the point wh	ere	(10)	If you think two planes will meet, what wou air traffic controller to do to avoid a collisio	•
Will the p	planes meet at the point when	e the two routes intersect?		(11)	Write the number that is the slope of the solid line representing WAL27.	
(7) If NO, wh	nich plane will arrive first?	NAL63		(12)	Write the number that is the slope of the dotted line representing NAL63.	
	ny seconds will it take this pla the point where the two route		secs	(13)	What information does the slope of the line each plane?	e tell you about
(Hint: At	me, how far away is the other that time, how many feet has ny feet is it from the intersecti	the second plane traveled?	feet			

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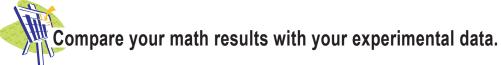
lr	nvestigator:	

## **ANALYZE YOUR RESULTS**





Your Task: Analyze and explain your results. Then apply what you learned to another problem.



1 Use your experimental data a	Experiment	Mathematics	Ose your ex	sperimental data and your math	
Did the planes meet where the routes meet? (Yes or No)			Separation (feet) where meet.	distance	
Do your experimental and you		natch?	Do your exp	perimental and your math results	s match?
If No, why do you think they o	don't match?		If No, why d	o you think they don't match?	
Which is correct? Why?			Which is co	rrect? Why?	

## **ANALYZE YOUR RESULTS**





## Compare the speeds and compare the distances.

V			
9 Are the planes' speeds the same or different?	Same	Different	
What is the "difference in the planes' starting dist (That is, what is the "headstart?")	ances?"	feet	
What is the difference in the planes' final position (What is the separation distance where the route		feet	•
Are these distances the same or different?	Same	Different	

Consider the general problem.

flying at th	e same spe	ec
nt routes.	The planes	3
erent dista	nces from	
ere the ro	utes meet.	1
	nt routes. erent dista	flying at the same spent of routes. The planes erent distances from ere the routes meet.

13	When the first plane reaches the point where	the
کرک	routes meet, the separation distance will be:	

the same as different than

the "difference" in the planes' starting distances (the "headstart").

Smart Skies™

ln١	/estiga	tor:		

## **EXTENSION**





Your Task: Will the two planes violate the separation standard where the routes meet?



For safety reasons, the planes must be separated by at least 5 feet at the point where the routes meet.



$\binom{1}{1}$	Based upon your calculation, what is the difference in the planes' final positions?  (That is, what is the planes' separation distance where the routes meet?)  feet
(2)	Does this distance satisfy the separation requirement?  Yes No
(3)	If NO, what would you tell the air traffic controller to do to meet the separation requirement?

Investigator:	

## **EXTENSION**



 To avoid a separation violation, the air traffic controller gives WAL27 a new route. Analyze an alternate route.

 The planes are traveling at the same speed.

WAL27 speed =  $\frac{1}{2}$  foot per second NAL63 speed =  $\frac{1}{2}$  foot per second

How far will WAL27 travel to reach the new point where the routes meet?

feet

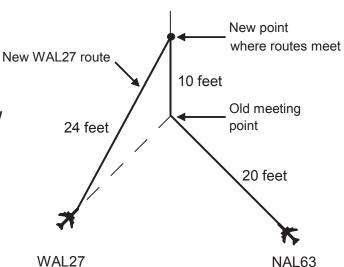
How far will NAL63 travel to reach the **new** point where the routes meet?

feet

Which plane will fly a shorter distance?

WAL27

NAL63



When WAL27 arrives at the **new** point where the routes meet, how many feet away is NAL63 on its route?

feet

Here's another way to think about this.

When WAL27 has traveled 24 feet to the **new** point where the routes meet, how far has NAL63 traveled?

feet

How much shorter?



At that time, how many feet away is NAL63 on its route?

feet

) Is the separation standard violated?

Yes

No

